ENGR 206

Experiment #3
Kirchhoff's Laws

Objective

To verify experimentally Kirchhoff's voltage and current laws, as well as the principles of voltage and current division.

Introduction

Kirchhoff's laws are the most widely used laws in circuit analysis. They are stated as follows:

- **Kirchhoff's Voltage Law (KVL)**: The sum of all voltages around a closed loop is zero.
- **Kirchhoff's Current Law (KCL)**: The sum of all currents for any node is zero.

In this laboratory, we will investigate KVL and KCL with a few experiments, starting with simple voltage and current dividers.

Voltage divider

A resistive voltage divider is a simple circuit comprising a voltage source and two resistors in series, as shown in Figure 1a.

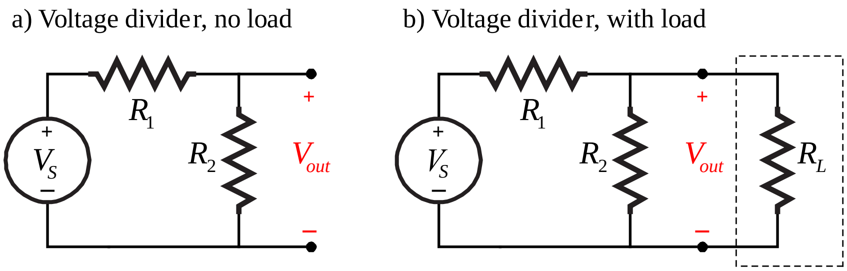

Figure 1: Voltage divider

The voltage divider is often used to supply an output voltage, V_{out}, that is lower than a source voltage, V_S. The source might be a voltage source, such as a battery or power supply, or by a piece of equipment such as an amplifier or other instrument. It's a simple circuit – simple to understand and to implement – but it has limitations, as we shall see.

First, let's analyze the circuit in the absence of an added load. From Figure 1a, application of KVL and KCL results in the voltage divider formula:

$$V_{out} = V_s \frac{R_2}{R_1 + R_2}. \tag{1}$$

As R_2 gets larger with respect to R_1 (i.e. for $R_2 >> R_1$), V_{out} tends towards V_s since

$$\lim_{R_2 \to \infty} \left(\frac{R_2}{R_1 + R_2} \right) = 1.$$

On the other hand, when $R_2 << R_1$, V_{out} tends towards zero. And when $R_2 = R_1$, then $V_{out} = V_s/2$.

If an additional load resistor, R_L, is interposed in parallel across R_2, then you can show that

$$V_{out} = V_s \frac{(R_2 \parallel R_L)}{R_1 + (R_2 \parallel R_L)} = V_s \frac{R_2 R_L}{R_1 R_2 + R_1 R_L + R_2 R_L} . \qquad (2)$$

Since you can show that the parallel combination of two resistors results in an equivalent resistance that is always lower than that of either resistor, the net effect of loading the voltage divider with R_L is to reduce V_{out} compared with the unloaded case of Figure 1a.

R-2R divider

An extension of the simple voltage divider is an $R-2R$ divider. A form of this circuit – called an $R-2R$ ladder – is at the heart of digital-to-analog (D/A) converters (also called DACs) that are found ubiquitously in digital electronic devices, for example, in a cell phone or digital music player, where they are used to convert digital signals – a series of digital words – into analog signals that can be amplified and played to a user. In a later lab we will explore the use of the $R-2R$ ladder in a D/A converter. For now, we'll use a ladder of resistors to form a clever voltage divider. Figure 2a shows an example of such a voltage divider built using an $R-2R$ ladder.

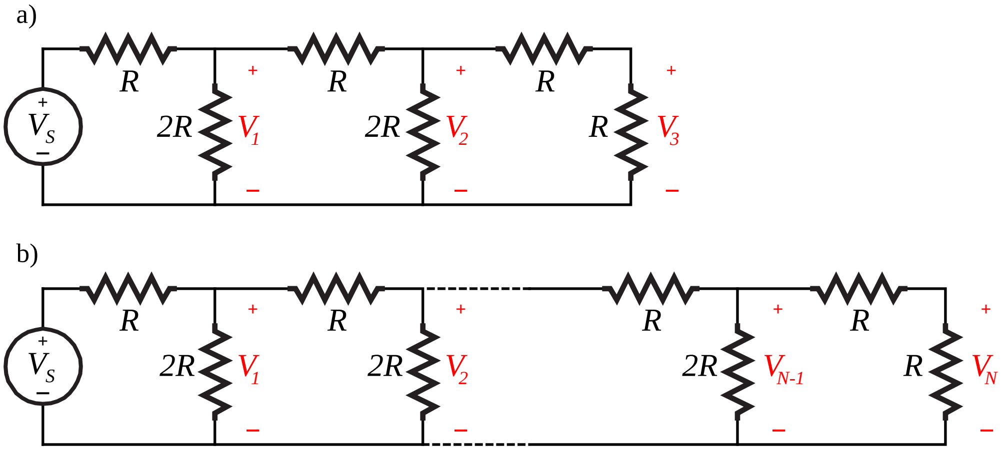

Figure 2: R-2R ladder

The name $R-2R$ ladder comes from the configuration of resistors. The three resistors that form the horizontal rungs running across the top of the ladder have value R, whereas the vertical resistors have value $2R$, except for the last (rightmost) resistor, which has value R. For this case, you can verify that the voltages across the vertical resistors are

$$V_1 = \frac{V_S}{2},\ V_2 = \frac{V_S}{4} \text{ and } V_3 = \frac{V_S}{8} . \qquad (3)$$

For the more general case of Figure 2b, which has N vertical resistors, you can verify that that the voltage across the k^{th} resistor is

$$V_k = \frac{V_S}{2^k}$$

Hence, the effect of $R-2R$ ladder is to create a series of voltages which differ from each other by a factor of two.

Current divider

A resistive current divider is a simple circuit comprising a current source and two resistors in parallel, as shown in Figure 3a.

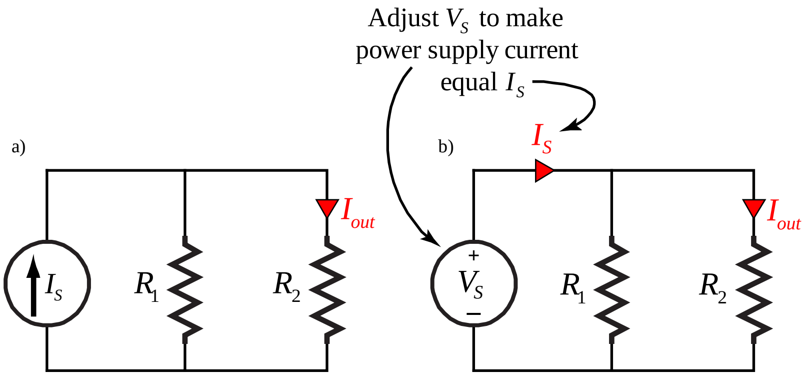

Figure 3: Current Divider

Just as the name implies, the current divider splits the current between the two resistors such that the current flowing through R_2 is

$$I_{out} = I_S \frac{R_1}{R_1 + R_2} . \tag{4}$$

Note that as R_2 goes towards zero, all the current will flow through R_2 and hence, I_{out} will tends towards I_S . Conversely, as R_2 tends towards $+\infty$, I_{out} will tend towards zero. Note also that by KCL, the sum of the currents in the two resistors must always be equal to I_S .

In the laboratory, we will use the Agilent E3631A Triple Output Power Supply to provide a constant current. Notice that the display of the supply shows both the voltage being selected by the voltage adjustment knobs (on the left side of the display) and the current being provided by the supply (on the right side). To emulate a current source, hook up the voltage supply as shown in Figure 3b, and then adjust the voltage until the current reads the desired value, I_S .

Potentiometer

A potentiometer (called a "pot" for short), is a variable resistor. Potentiometers come in many shapes and sizes, including dial, slider and trimpot, and use many different resistive materials, including wire and thin films. But the function of all potentiometers is the same: to provide a resistance whose value can be manually adjusted by the user over some defined range. You are familiar with the use of potentiometers as volume controls in old-school electronic equipment, such as your voltage supply. A picture of typical potentiometer is shown in Figure 4a. A mechanical schematic is shown in Figure 4b and an electrical symbol of a potentiometer is shown in Figure 4c.

There are three terminals, labeled A, W and B. Terminals A and B are connected to the two ends of resistive material. The resistance between these terminals is constant and represents the maximum total resistance the potentiometer is capable of providing, for example $10\,K\Omega$. Terminal W is connected to a wiper that turns with the shaft of the potentiometer. The wiper contacts the resistive material at positions between A and B. Hence, the resistance measured between A and W or between W and B is always a fraction of the maximum resistance. In a linear pot, resistance varies directly with the rotation of the knob. In the example shown in Figure 4d, when the wiper on a linear $10\,k\Omega$ pot is turned clockwise 80% of its maximum range, the resistance between A and W is $8\,k\Omega$ and the resistance between W and B is equal to the total resistance minus the resistance between A and W, namely $2\,k\Omega$.

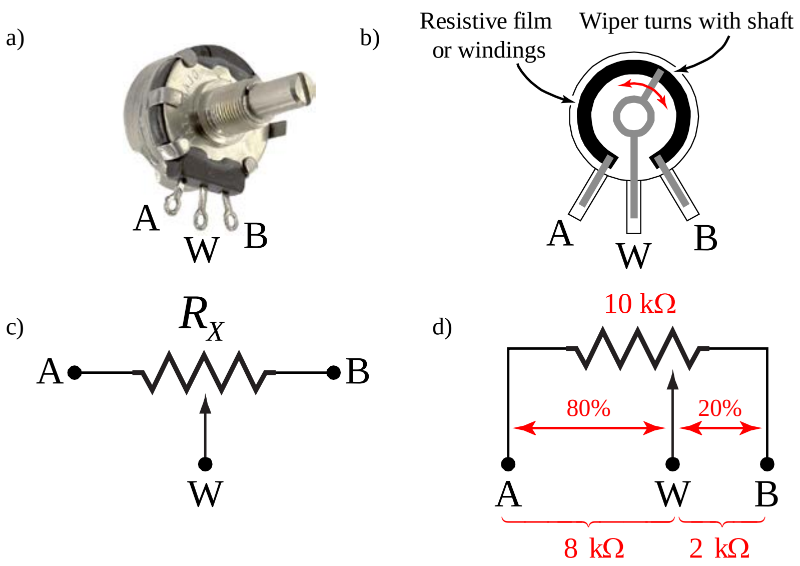

Figure 4: Potentiometer and its equivalent circuit.

A potentiometer can be used as a variable resistor or as a variable voltage divider. When used as a variable resistor, only two terminals are used: W and either A or B. In this configuration, it is common practice to connect the unused terminal to the wiper to eliminate possible noise pickup. When used as a voltage divider, all three terminals are used, as shown in Figure 5.

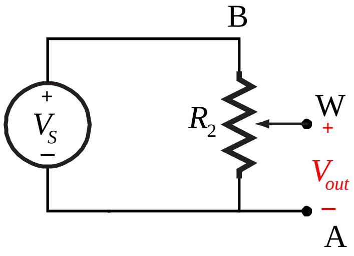

Figure 5: Using a potentiometer as a voltage divider

Wheatstone bridge

A Wheatstone bridge is a circuit that is used to measure resistance, capacitance and inductance. In this laboratory, we will investigate the a bridge designed to measure the value of an unknown resistor. The basic resistive bridge circuit is shown in Figure 6a. There are four resistors and a voltage source. Given a constant voltage, V_S , you can show that the output voltage, V_{out} , is given by

$$V_{out} = V_S\left(\frac{R_3}{R_1+R_3} - \frac{R_x}{R_2+R_x}\right) = V_S\left(\frac{R_2R_3 - R_1R_x}{(R_1+R_3)(R_2+R_x)}\right). \tag{5}$$

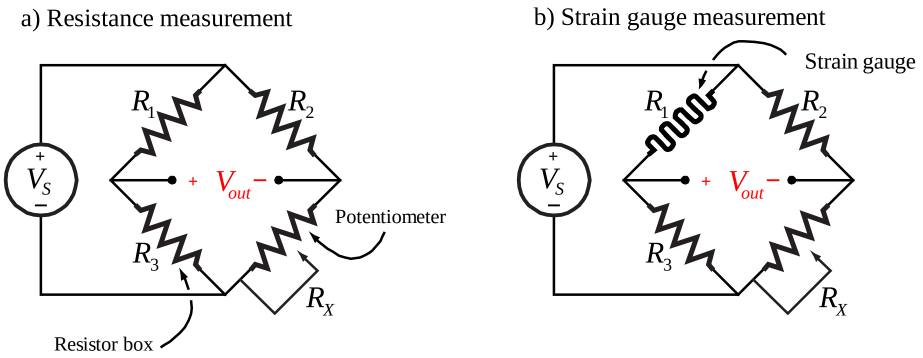

Figure 6: Resistive Wheatstone bridge

If the output voltage, $V_{out} = 0$, the bridge is said to be in balance. You can verify that when the bridge is balanced,

$$\frac{R_1}{R_3} = \frac{R_2}{R_x}, \tag{6a}$$

The resistive bridge can be used to measure the value of an unknown resistor (e.g. R_x) by fixing the values of resistors R_1 and R_2 and then adjusting resistor R_3 using a "decade box" found in the laboratory until the bridge is in balance. Then,

$$R_x = \frac{R_2 R_3}{R_1}. \tag{6b}$$

The bridge can also be used to measure changes in resistance produced by a sensor such as a photocell or a strain gauge. Figure 6b shows the experimental setup where constant resistor, R_1, is replaced by a strain gauge, a device whose resistance is proportional to applied force.

Prelaboratory Work

Derive Equations (1) through (6).

Laboratory Work

1. Voltage divider (unloaded). The purpose of this section is to verify the voltage divider formula experimentally.
   a. Construct the voltage divider circuit of Figure 1a with $R_1 = R_2 = 1k\Omega$.
   b. Turn the power supply on. With the help of the DC voltmeter, adjust the power supply to output +10 V.
   c. Measure the voltage around the loop and verify the validity of KVL for this particular circuit.
   d. Compare the measured V_{out} with the theoretically computed V_{out}. Account for causes of possible discrepancy.
   e. Leave R_2 in place, repeat steps 4 and 5 for the cases $R_1 = 10\,k\Omega$ and $R_1 = 100\,k\Omega$.
2. Voltage divider (loaded). The purpose of this section is to understand the effect of loading on a voltage divider.
   a. With $V_S = 10V$, $R_1 = R_2 = 1k\Omega$, attach a load resistor $R_L = 1k\Omega$ resistor in parallel, as shown in Figure 1b and measure V_{out}.
   b. Repeat with $R_L = 10\,k\Omega$ and $R_L = 100\,k\Omega$. Compare your measured results to the theoretical predictions using these resistor values. What can you conclude about the effects loading the output of a voltage divider with a load resistor?
   c. Variable voltage divider
   d. Connect the $10\,k\Omega$ potentiometer in your kit to the power supply to form a variable voltage divider, as shown in Figure 5. Make sure that terminals A and B are across the voltage source, not A and W or B and W!
   e. With $V_S = 10V$, monitor V_{out} with the DC voltmeter as you rotate the dial of the pot back and forth. Within what limits does V_{out} vary? Why?
3. R-2R divider.
   a. Connect the $R-2R$ divider as shown in Figure 2a using $V_S = +10V$ and $R = 1k\Omega$.

    b. Measure the outputs, V_1, V_2 and V_3. Do they match the theoretical prediction?

4. Current Division. The purpose of this section is to verify the current divider formula experimentally.
    a. Construct the voltage divider circuit as shown in Figure 3b using the output of the 0 to 6 volt power supply output. Use $R_1 = R_2 = 1k\Omega$. Make sure that the power supply output is off while constructing the circuit.
    b. Turn the output of the supply on and adjust the output voltage until the measured current, I_S, reaches 10mA.
    c. Use the DC ammeter to verify KCL by measuring the current, I_{out}, through the resistors R_1 and R_2.
    d. Compare your measured values with the theoretical values found using current divider formula. Account for the causes of possible disagreements.
    e. Leave R_2 in place, and repeat steps c) and d) for the case where $R_1 = 5.1k\Omega$ and for the case where $R_1 = 10\,k\Omega$. Be sure to maintain a constant current of 10mA as viewed on the power supply display.
5. Bridge Circuit. The purpose of this section is to understand how a bridge circuit can be used to measure an unknown resistance.
    a. Construct the bridge of Figure 6a using $R_1 = 2.2\,k\Omega$, $R_2 = 4.7\,k\Omega$, a decade resistor box for R_3 and the potentiometer for R_x. Set the decade box to about $3\,k\Omega$ and the pot dial position about $1/3$ way of its full range.
    b. Use the constant voltage output of the +25V supply, and adjust the output to +10V.
    c. Monitor the output voltage, V_{out}, with a DC voltmeter. Adjust the decade box resistance value until the voltmeter reading is as close to 0 as possible. Using Equation (6b), compute R_x
    d. Leave the pot on the breadboard but carefully disconnect it electrically from the circuit (i.e. remove its connection to R_2) without changing its shaft position. Now measure the resistance, R_x, using the ohmmeter and compare with the value obtained in the previous step. Discuss the causes of possible discrepancies.