ENGR 206

Experiment #5
Thévenin Equivalent Circuits

Objective

To construct Thévenin equivalent circuits and verify the maximum power transfer theorem.

Introduction

A linear network is a network that consists only of a combination of resistors and independent or dependent voltage and current sources. A one-port network is one that interfaces with the world through a single pair of terminals, as shown in Figure 1a. Thévenin's theorem states that any linear, one-port electrical network with two terminals is electrically equivalent to a single ideal voltage source, V_{TH} , in series with a single resistor, R_{TH} , as shown in Figure 1b. Norton's theorem states that such a network is also electrically equivalent to a single ideal current source, I_N , in parallel with a single resistor, R_N (shown in Figure 1c).

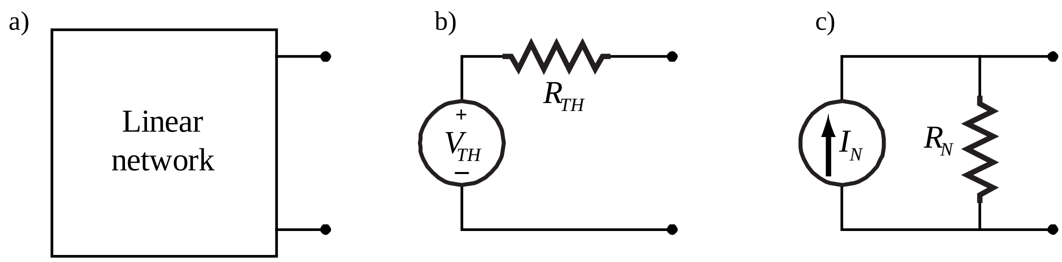

Figure 1

These two theorems provide a powerful way to simplify the analysis of circuits. For example, by replacing a large and complicated network of elements by its Thévenin equivalent, one can rapidly calculate the maximum power that the circuit can deliver to a load.

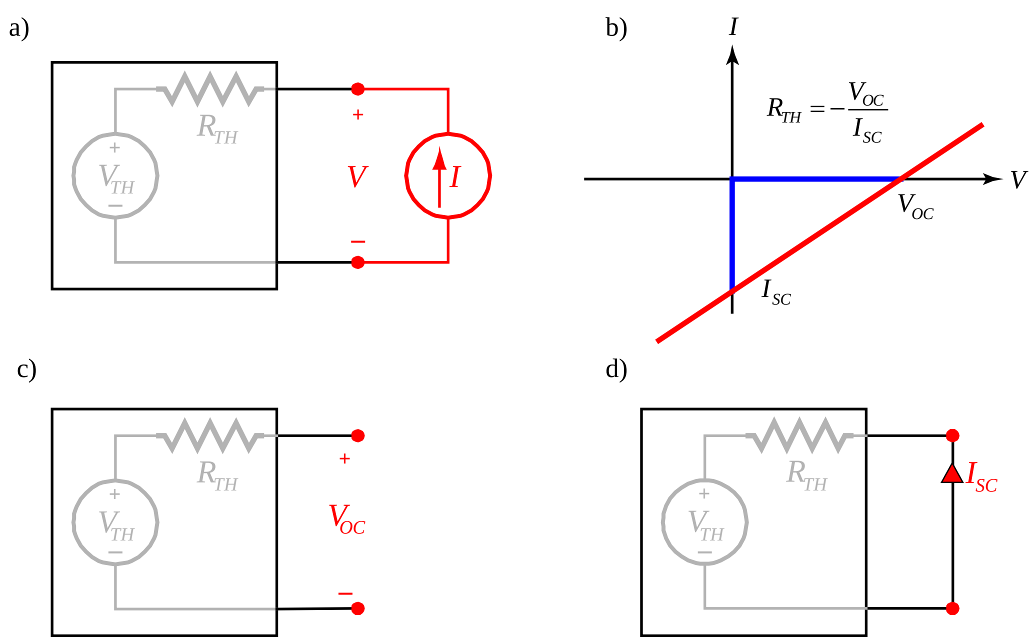

Figure 2

In this laboratory, we will use theoretical analysis, PSpice simulation and experimental measurements to verify Thévenin's theorem.

Thévenin equivalent circuits

In order to understand equivalent circuits, consider the linear one-port network shown in Figure 2a. One way to characterize this network is to measure the voltage, *V*, induced across the terminals in response to injecting a current, *I*, into the network. You should be able to show (for example by using the principle of superposition) that the voltage is linearly related to the current:

$$V = V_{TH} + I\,R_{TH}\,. \qquad (1)$$

The resulting current-voltage (*I*-*V*) response, shown in Figure 2b, is a straight line that is completely specified by two points: the *open-circuit voltage*, V_{OC}, and the *short-circuit current*, I_{SC}. The open-circuit voltage is the voltage across the terminals when the terminals are unconnected to anything else, i.e., when $I = 0$, as shown in Figure 2c. When $I = 0$, no current flows through R_{TH}, so $V_{OC} = V_{TH}$. The short-circuit current is the current that flows into the circuit through the terminals when they are connected together, i.e., when $V = 0$, as shown in Figure 2d. Solving Equation (1) for $V = 0$ gives

$$I_{SC} = -\frac{V_{TH}}{R_{TH}},$$

so, given measurements of $V_{OC} = V_{TH}$ and I_{SC}, we can compute

$$R_{TH} = -\frac{V_{OC}}{I_{SC}}, \qquad (2)$$

which is just the slope of the *I*-*V* response.

Figure 3a shows an example of how the Thévenin equivalent of a circuit with multiple sources can be derived:

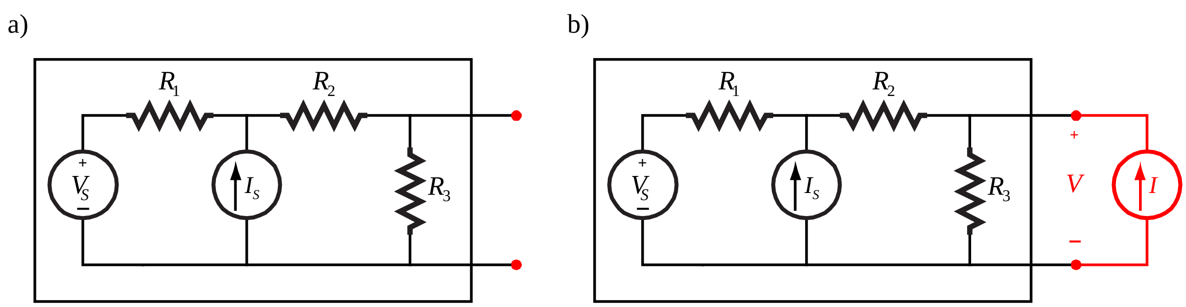

Figure 3

As suggested in Figure 2a, we can hook a current source up to the terminals and measure the output voltage, *V*, as a function of the injected current, *I*. We can use the superposition principle to split this into three sub-problems, as shown in Figure 4. In each sub-problem, we suppress all but one source. In Figure 4a, we suppress all sources except V_S; in Figure 4b, we suppress all but I_S; in Figure 4c, we suppress all the internal sources of the network and leave only the external current source, *I*.

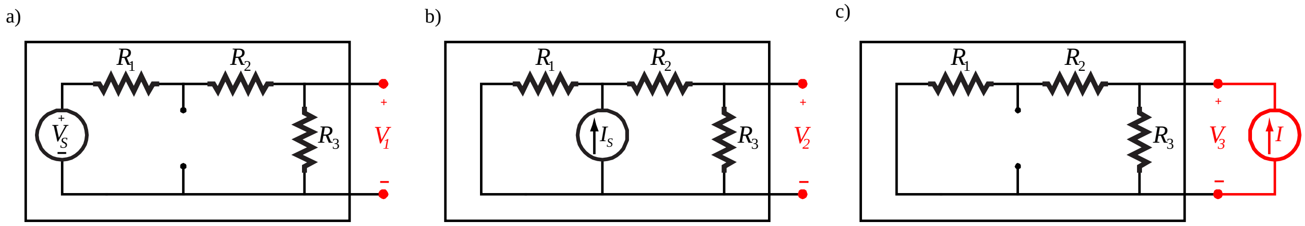

Figure 4

You should be able to verify that

$$V = V_1 + V_2 + V_3 = \underbrace{\left(V_S \frac{R_3}{R_1 + R_2 + R_3}\right)}_{V_1} + \underbrace{\left(I_S \frac{R_1 R_3}{R_1 + R_2 + R_3}\right)}_{V_2} + \underbrace{\left(I \frac{(R_1 + R_2) R_3}{R_1 + R_2 + R_3}\right)}_{V_3} \quad (3)$$

Comparing this equation to Equation (1), you can see that the first two terms are due to the internal sources and do not depend on the externally injected current. Their sum, $V_1 + V_2$, is the open-circuit voltage. The last term is due to the external current and does not depend on the internal sources, and is equal to IR_{TH}. So we have determined that

$$V_{TH} = V_{OC} = \frac{(V_s + R_1 I_S) R_3}{R_1 + R_2 + R_3} \text{ and } R_{TH} = \frac{(R_1 + R_2) R_3}{R_1 + R_2 + R_3}.$$

You can see that in this case (where there are only independent sources), the Thévenin-equivalent resistance is just $R_3 \| (R_1 + R_2)$, which is the resistance you would get by suppressing all the internal sources of the circuit and measuring the resistance across the terminals.

In order to measure the Thévenin-equivalent resistance, we can also calculate the open-circuit voltage, V_{OC}, and the short-circuit current, I_{SC}, and then apply Equation (2). We've already calculated the open-circuit voltage using superposition in Figure 4a and b. Figure 5 shows how we could calculate I_{SC}. Using the principle of superposition, we could determine the components of the short-circuit current, I_1 and I_2, that flow due to each independent source acting alone.

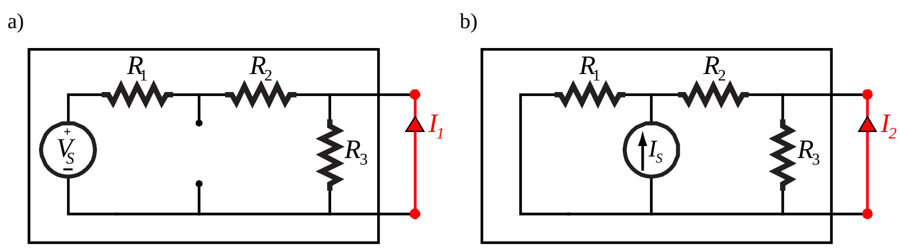

Figure 5

You can verify that

$$I_{SC} = I_1 + I_2 = \underbrace{-\frac{V_S}{R_1 + R_2}}_{I_1} \underbrace{-\frac{I_S R_1}{R_1 + R_2}}_{I_2} = -\frac{V_S + I_S R_1}{R_1 + R_2}.$$

Then, by Equation (2),

$$R_{TH} = -\frac{V_{OC}}{I_{SC}} = -\frac{\dfrac{(V_s + I_S R_1) R_3}{R_1 + R_2 + R_3}}{-\dfrac{V_S + I_S R_1}{R_1 + R_2}} = \frac{(R_1 + R_2) R_3}{R_1 + R_2 + R_3},$$

which is the same answer we got before. As a practical matter in this laboratory, we will just measure V_{OC} and I_{SC} and from these measurements calculate R_{TH}.

Maximum power transfer

Consider the circuit of Figure 6. We've attached a load resistor with value, R_L , to a circuit which is modeled by it's Thévenin equivalent.

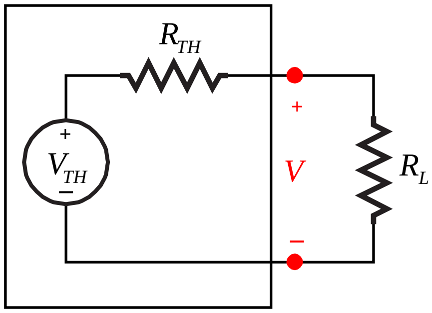

Figure 6

What is the value of R_L that maximizes the power transferred to the load by the source? Given that the voltage across the load resistor is

$$V = V_S \frac{R_L}{R_{TH} + R_L},$$

the power consumed by the load is

$$P = \frac{V^2}{R_L} = V_S^2 \frac{R_L}{(R_{TH} + R_L)^2}.$$

The maximum power transfer occurs when the derivative of P with respect to R_L is zero. You can verify that this occurs when

$$R_L = R_{TH}.$$

In this laboratory we will verify Thévenin's theorem on several circuits, and we will use the theorem to verify the maximum power that can be delivered by two terminals of a circuit across a load resistor.

Pre-laboratory work

1) Review Thévenin's and Norton's theorems.
2) Find the Thévenin equivalent circuits – that is, V_{TH} and R_{TH} – for each of the circuits in Figure 7, Figure 8 and Figure 9.
3) Using PSpice, create a simulation of each circuit described in the 'Experimental Work' section, below. This way, you'll understand what you need to do for your experimental work when you come to the laboratory, and you'll
4) be able to compare the results of your experiments with the PSpice simulations. Note that if you attempt to find the open-circuit voltage of the circuits of Figure 7 or Figure 8, PSpice will give an error, because in each case resistor R_3 is not connected to anything. To measure the open-circuit voltage, you either have to remove R_3 temporarily from your schematic, or temporarily attach an enormous resistor (e.g. $1G\Omega$) across the terminals.

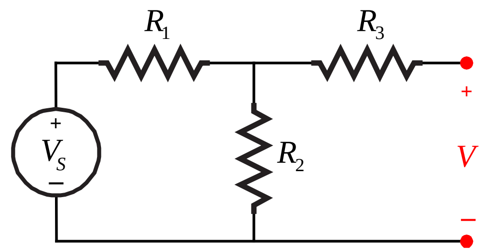

Figure 7

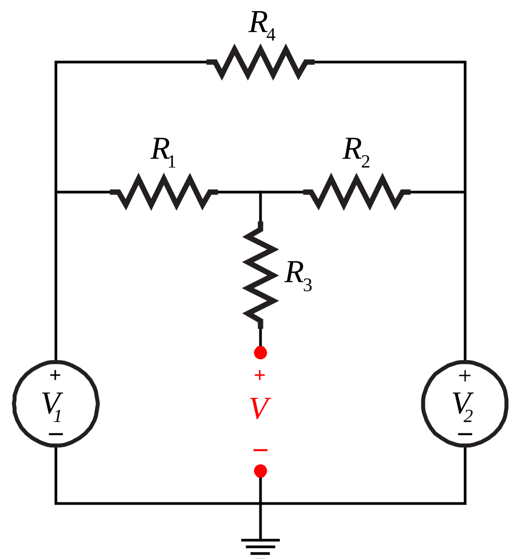

Figure 8

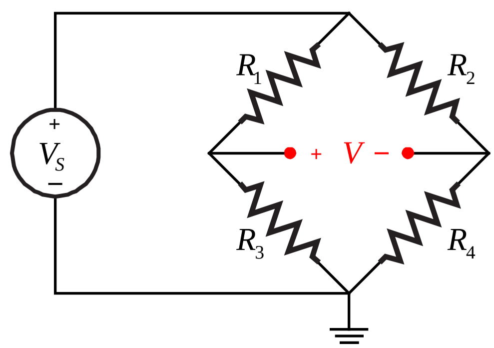

Figure 9

Laboratory work

1) Thévenin equivalent, one source. The purpose of this part is to find the Thévenin equivalent of a circuit with one source by measuring find V_{OC} and I_{SC}.
   a) Construct the circuit of Figure 7. Choose $R_1 = R_2 = R_3 = 10k\Omega$ and set the value of $V_s = 6V$ using the 10V supply.
   b) Measure the open-circuit voltage, V_{OC}, and the short-circuit current, I_{SC}, and thereby find V_{TH} and R_{TH}. Note: Do not attempt to measure R_{TH} directly with the ohmmeter! Explain in your report why this is a bad idea.
   c) Using PSpice, create a simulation of the circuit. Attach a current source (IDC on PSpice's list of sources) to the terminals of the circuit, in a manner similar to what we did in Figure 4, and do a DC sweep of the current from at least -3 mA to 3 mA. Consult the tutorial for instructions on doing such a sweep. Plot the I-V response, as we did in Figure 2b, and include it in your laboratory report. From this plot, read off values of V_{OC} and I_{SC}, and compare these to your theoretical calculations and the experimental measurements.
   d) Now we'll check the values of V_{TH} and I_{TH} by injecting a current into the circuit and measuring the voltage, as in Figure 2a. We'll use the +25V voltage source and measure the voltage it takes to inject a given amount of current into the circuit, as we did in Experiment #2. So, leaving in place the 10V supply in place for V_S, connect the +25V supply to the terminals of the circuit read the current flowing into the circuit from the power supply's display as you adjust the voltage. Measure the voltage it takes to inject 2 ma of current into the circuit. Repeat with 0 ma. Do these points lie along the I-V curve you plotted in part 1b?
2) Thévenin equivalent, two sources. The purpose of this part is to find the Thévenin equivalent of a circuit with two voltage sources and verify that you have the right values of V_{TH} and R_{TH} by constructing the equivalent circuit with those values.
   a) Construct the circuit of Figure 8 using the values $V_1 = -5V$ (note the minus sign!), $V_2 = 10V$, $R_1 = 5.1k\Omega$, $R_2 = 2.2k\Omega$, $R_3 = 3.3k\Omega$ and $R_4 = 6.8k\Omega$.
   b) Measure the open-circuit voltage, V_{OC}, and the short-circuit current, I_{SC}, the same way you did in part 1b and thereby find V_{TH} and R_{TH}. Plot a theoretical I-V response curve.
   c) Connect a $1k\Omega$ load resistor across the output terminals of the circuit and measure the resulting voltage across the terminals. Also measure the current through the load resistor. Repeat the measurement of voltage and current using a $10k\Omega$ load resistor. Plot these two points on your I-V graph in part a). Do they fit the characteristic? Discuss.
   d) Construct the Thévenin equivalent of the circuit of Figure 8. You can leave the circuit of Figure 8 in place if you like (for reference) and create the second circuit another part of your circuit board using the 6V supply for V_{TH} and the decade resistor box for R_{TH}. Find V_{OC} and I_{SC} for this circuit. Do they agree with those for the original circuit?
   e) Repeat step 2c for the Thévenin equivalent circuit. How closely do the resulting I-V values correspond to those of the original circuit?
3) Thévenin equivalent, voltage and current sources. The purpose of this part is to find the Thévenin equivalent of a circuit which contains both voltage and current sources.
   a) Construct the circuit of Figure 3a using the values $V_1 = -10V$, , $R_1 = 1k\Omega$, $R_2 = 2.2k\Omega$ and $R_3 = 1k\Omega$. Configure the +6V source as the current source, as you have done in previous laboratory experiments.
   b) Measure the open-circuit voltage, V_{OC}, and the short-circuit current, I_{SC}, the same way you did in part 1b and thereby find V_{TH} and R_{TH}. Plot the theoretical I-V response curve.
   c) Now, hook up the +25V source as a current source to the output terminals of your circuit, as shown by the red source in Figure 3b. Plot the voltage across the terminals for current values 0, 2, 4, 6, 8 and 10 mA on the graph you did in part b). Do the results agree with those of part b)?
4) Maximum power. In this part of the laboratory, we will use the Thévenin equivalent to predict the value of a load resistor that draws the maximum power from a circuit, and then verify this via simulation and experiment.
   a) Consider the circuit of Figure 9 with $V_s = 6V$, $R_1 = 1k\Omega$, $R_2 = 3.3k\Omega$, $R_3 = 2.2k\Omega$ and $R_4 = 1k\Omega$. By any method you choose (i.e. theoretical analysis or PSpice simulation), find V_{TH} and , and hence, the value of a load resistor that will extract maximum power from the circuit.

b) Construct the circuit on your breadboard. Measure the open-circuit voltage, V_{OC}, and the short-circuit current, I_{SC}, and thereby find V_{TH} and R_{TH}. Do the values agree with those in part a?

c) Attach the decade resistor box across the terminals of your circuit to act as a load resistor, R_L. Measure the voltage across the load (or the current through the load, if you wish) as you vary the resistance in $100\,\Omega$ increments over a range of about 50% of R_{TH} to 200% of R_{TH}. Plot the power consumed by the experimental circuit at each value of R_L. From the graph, find the value of R_L that corresponds to the maximum power point.

d) Using PSpice, create a simulation of the circuit with a load resistor, R_L, across the terminals with a value '{Rload}', as shown in Figure 10 (note the squiggly brackets).

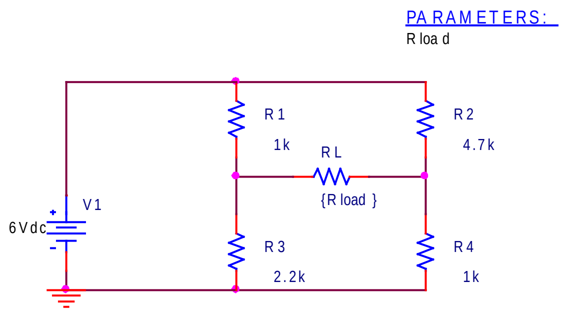

Figure 10

e) Do a parametric sweep of R_L from about $100\,\Omega$ to $10\,k\Omega$ in $100\,\Omega$ increments. Consult the tutorial for instructions on doing a parametric sweep. Now plot the power consumed by R_L. You can do this as follows: After you have run the simulation, in the probe window: Trace→Add Trace… When the 'Add Traces' panel pops up, enter into the 'Trace Expression' box at the bottom: 'I(RL)*I(RL)*Rload'. That plots the power ($P = I^2 R$)! From the graph, determine the value of R_L that corresponds to the maximum power consumed by the load. Does this value agree with the theoretically predicted and experimentally measured values? Explain reasons for possible discrepancies.