# ENGR 206 <br> Experiment 8 <br> Operational Amplifiers, II 

## Objective

To experiment with operational amplifiers connected in summing and differential configurations. To understand negative resistance and current-to-voltage conversion.

## Introduction

In the previous laboratory, we investigated basic inverting and non-inverting amplifier circuits using operational amplifiers (op amps). In this laboratory, we'll look at a number of further applications that exploit the capabilities of these remarkable circuit elements.


Figure 1: Summing and differential amplifier

## Summing amplifier

Figure 1a shows the circuit of an op amp configured as a summing amplifier. The output of the amplifier is proportional to the sum of the $N$ inputs, $v_{1}, \cdots v_{N}$. Notice that the summing amplifier is basically just the inverting amplifier with more than one input connected to the inverting terminal. The analysis of this amplifier configuration is identical to that of the inverting amplifier. Since no current flows into the terminals of the op amp, the non-inverting terminal is at ground, which makes the inverting terminal a "virtual ground". Applying KCL at the inverting terminal gives

$$
\frac{v_{1}}{R_{1}}+\cdots+\frac{v_{N}}{R_{N}}+\frac{v_{\text {out }}}{R_{F}}=0 .
$$

So,

$$
v_{\text {out }}=-\frac{R_{F}}{R_{1}} v_{1}-\cdots-\frac{R_{F}}{R_{N}} v_{N}=-R_{F}\left(\frac{v_{1}}{R_{1}}+\cdots+\frac{v_{N}}{R_{N}}\right)
$$

We attach a resistor of value $R_{\chi}=\left(R_{1}\left\|\cdots R_{N}\right\| R_{F}\right)$ to the non-inverting terminal to compensate for the input offset current, an effect we discussed in the last laboratory.
When the input resistors, $R_{1}, \cdots, R_{N}$ have different values, this circuit provides the sum of the individually scaled inputs. In the particular case in which all the input resistors are equal, $R_{i n} \triangleq R_{1}=\cdots=R_{N}$, then the output is scaled sum of the inputs,

$$
v_{\text {out }}=-\frac{R_{F}}{R_{\text {in }}}\left(v_{1}+\cdots+v_{N}\right),
$$

and further, if $R_{F}=R_{i n}$, then the output is just a unity-gain sum of the inputs.

Each input of the summing amplifier is effectively isolated from the other inputs, because even though all the inputs are tied together through their input resistors at the inverting terminal, the inverting terminal is effectively at ground. Summing amplifiers are useful in many applications, including audio mixers, and digital-to-analog (D/A) converters.

## Differential amplifier

The differential amplifier, shown in Figure 1b, produces an output that is proportional to the difference of two input signals. You can view it as a hybrid of the inverting and non-inverting amplifiers, since inputs are presented to both terminals. At the non-inverting terminal,

$$
v_{+}=v_{2} \frac{R_{4}}{R_{3}+R_{4}},
$$

and by KCL at the inverting terminal,

$$
\frac{v_{1}-v_{-}}{R_{1}}+\frac{v_{\text {out }}-v_{-}}{R_{2}}=0
$$

By the golden rules of op amps, $v_{-}=v_{+}$. A bit of algebra yields,

$$
\begin{align*}
v_{\text {out }} & =v_{2}\left(\frac{R_{4}}{R_{3}+R_{4}}\right)\left(\frac{R_{1}+R_{2}}{R_{1}}\right)-v_{1} \frac{R_{2}}{R_{1}}=v_{2}\left(\frac{1+R_{2} / R_{1}}{1+R_{3} / R_{4}}\right)-v_{1} \frac{R_{2}}{R_{1}} .  \tag{1}\\
& =v_{2} \frac{R_{2}}{R_{1}}\left(\frac{1+R_{1} / R_{2}}{1+R_{3} / R_{4}}\right)-v_{1} \frac{R_{2}}{R_{1}}
\end{align*}
$$

In the case where $R_{1} / R_{2}=R_{3} / R_{4}$, then the output of the circuit is the scaled difference between the two inputs:

$$
v_{\text {out }}=\frac{R_{2}}{R_{1}}\left(v_{2}-v_{1}\right),
$$

When $R_{1}=R_{2}=R_{3}=R_{4}$, the circuit is a unity gain difference "amplifier": $v_{\text {out }}=v_{2}-v_{1}$.

Note that if $v_{1}$ is grounded, the differential amplifier reduces to a non-inverting amplifier (with a voltage divider at the input). If $v_{2}$ is grounded, the differential amplifier reduces to an inverting amplifier.

## Negative resistance

An op amp circuit can be set up to look like a negative resistance. Consider the circuit of Figure 2a.


Figure 2: Negative resistance converter
It is basically a non-inverting amplifier with an additional resistor, $R_{3}$, between the $v_{\text {out }}$ and $v_{\text {in }}$. In order to find the Thévenin equivalent of the circuit in the dotted box, we hook up a voltage source, $v_{s}$, and measure the current flowing into the circuit, is. Here, $v_{i n}=v_{s}$, so for a non-inverting amplifier, the output is

$$
v_{\text {out }}=v_{s}\left(1+\frac{R_{2}}{R_{1}}\right),
$$

and by KCL at the non-inverting terminal,

$$
i_{s}=\frac{v_{s}-v_{\text {out }}}{R_{3}}=v_{s} \frac{-\left(\frac{R_{2}}{R_{1}}\right)}{R_{3}} .
$$

Hence, the circuit in the box is equivalent to a negative resistor of value

$$
R_{e q}=\frac{v_{s}}{i_{s}}=-\frac{R_{1}}{R_{2}} R_{3} .
$$

If we select $R_{1}=R_{2}$, then $R_{e q}=-R_{3}$.
For any resistor with voltage, $V$, across it, the power dissipated is $V^{2} / R$. For a normal passive resistor, $R$ is positive, so power is always being consumed by the resistor. However, in this circuit, $R$ is negative, which means the power is being supplied to the circuit by the "resistor" rather than dissipated!

## Voltage-controller current source

Figure 3a shows a key application of the negative resistance converter: a voltage-controlled constant-current source.


Figure 3: Voltage-controlled current source

The purpose of a current source is to provide a constant current, $i_{L}$, to a load, which we model by a resistor of value $R_{L}$. Constant current means that $i_{L}$ is independent of the value of $R_{L}$. To analyze this circuit, replace the circuit in the box by its equivalent negative resistor, as shown in Figure 3b. Now, $v_{L}$, the voltage across $R_{L}$ is given by a simple voltage divider formed by $R_{S}$ and the parallel combination of the negative resistor and $R_{L}$ :

$$
v_{L}=v_{S} \frac{R_{L} \|\left(-\frac{R_{1}}{R_{2}} R_{3}\right)}{R_{S}+R_{L} \|\left(-\frac{R_{1}}{R_{2}} R_{3}\right)}
$$

A bit of algebra gives

$$
i_{L}=\frac{v_{L}}{R_{L}}=v_{S} \frac{1}{R_{S}+R_{L}\left(1-\frac{R_{2} R_{S}}{R_{1} R_{3}}\right)}
$$

In order that $i_{L}$ be independent of the load, $R_{L}$, we require

$$
\frac{R_{2} R_{S}}{R_{1} R_{3}}=1 \text { or } \frac{R_{S}}{R_{3}}=\frac{R_{2}}{R_{1}}
$$

Then the value of $i_{L}$ is determined only by the value of voltage source, $v_{S}$ and series resistor, $R_{S}$ :

$$
i_{L}=\frac{v_{S}}{R_{S}}
$$

so the equivalent circuit is a voltage-controlled constant-current source, as shown in Figure 3c. If we've already selected $R_{1}=R_{2}$, then we only need choose $R_{3}=R_{S}$.

## Current-to-voltage converter

The current-to-voltage (I-V) converter can be viewed as a simple extension of the inverting op amp and is shown in Figure 4a.


Figure 4: I-V converter
Since the inverting terminal is a virtual ground, solving KCL yields

$$
v_{\text {out }}=-i_{s} R .
$$

Figure 4b shows an application of the I-V converter. In this case, the source of current is a phototransistor, which basically converts incident light into a relatively small current. In this laboratory, we'll be using a phototransistor that is sensitive to light in the visible range. The data sheet is here.

## Laboratory work

1. Set up op amp. Turn on the power supply and adjust the $\pm 25 \mathrm{~V}$ outputs for $\pm 15 \mathrm{~V}$, then turn the power off while you connect the op amp. As a reminder, Figure 5a shows the pin configuration of the 741 op amp and Figure 5b shows the tidy way to connect the op amp to power and ground on your circuit board. Remember to bypass the $\pm 15 \mathrm{~V}$ supplies on the circuit board with $0.1 \mu \mathrm{~F}$ capacitors to ground.


Figure 5: Op amp connections
2. Summing amplifier.
a. Assemble the summing amplifier circuit shown in Figure 1a with only two inputs. Let $R_{1}=1 \mathrm{~K} \Omega$, $R_{2}=3.3 \mathrm{~K} \Omega, R_{F}=10 \mathrm{~K} \Omega$ and $R_{X} \approx R_{1}\left\|R_{2}\right\| R_{F}=2.2 \mathrm{~K} \Omega$.
b. First, we will measure the gain of the amplifier with each input active separately. Connect input $v_{1}$ to a $1-\mathrm{kHz}$ sinusoidal AC signal of 1 V peak amplitude and 0 V DC offset obtained from the function generator. Connect input $v_{2}$ to ground. Put the AC input on Ch1 of the scope and $v_{\text {out }}$ on Ch2 and make sure both channels are DC coupled. Set the scope to trigger on Ch1, AC coupled. Measure the output, $v_{\text {out }}$, and calculate the gain for this input. Now, connect input $v_{1}$ to ground and connect input $v_{2}$ to the AC signal. Again, measure the output, $v_{\text {out }}$, and calculate the gain for this input. Do the gains you have measured make sense given the values of the resistors?
c. Now we determine the output of the amplifier with two inputs connected. Connect input $v_{1}$ to the 1 V peak AC signal and connect input $v_{2}$ to a +2 V DC voltage obtained from the +6 V supply the function generator. Measure the peak-to-peak and mean value of the output. Does the value you obtain correspond to what you expect?
d. Slowly increase the DC offset of the function generator and note the value at which $v_{\text {out }}$ begins to saturate. Does this value make sense? Now decrease the DC offset and again note the value at which $v_{\text {out }}$ begins to saturate. Does this value make sense?
e. Adjust the DC offset of the function generator to make $v_{\text {out }}$ have a zero DC value (i.e. a mean value of zero). Does the value of the DC offset correspond to what you expect? Return the DC offset of the function generator to zero.
3. Difference amplifier.
a. Assemble the difference amplifier circuit shown in Figure 1b with $R_{1}=1 \mathrm{~K} \Omega, R_{2}=4.7 \mathrm{~K} \Omega, R_{3}=6.8 \mathrm{~K} \Omega$ and $R_{4}=3.3 \mathrm{~K} \Omega$.
b. We are now going to measure the gains of each input separately. Connect a $1-\mathrm{kHz}$ sinusoidal AC signal of 1 V peak amplitude and 0 V DC offset to input $v_{1}$ and ground input $v_{2}$. Measure the gain of the op amp. Does it match your expectation given the values of the resistors?
c. Now reverse the inputs: connect the AC signal to input $v_{2}$ and ground input $v_{1}$. Again measure the gain of the op amp. Does it match your expectation?
d. Now we connect both inputs. With the AC signal still connected to input $v_{2}$, connect a 1 V DC signal to input $v_{1}$. Explain your output. Save a picture of the display for your report.
e. Leave resistors $R_{1}=1 \mathrm{~K} \Omega$ and $R_{2}=4.7 \mathrm{~K} \Omega$ in place. But in the place of $R_{3}$ and $R_{4}$, use a $10 \mathrm{~K} \Omega$ potentiometer, as shown in Figure 6.
f. Attach the AC signal to both inputs of the amplifier. Adjust the wiper of the potentiometer so that the output of the amplifier is zero. Now carefully remove the connections from the potentiometer without disturbing the wiper position and measure both $R_{3}$ and $R_{4}$ with the ohmmeter. What is the relation between $R_{1} / R_{2}$ to $R_{3} / R_{4}$ and why?


Figure 6: Difference amplifier
4. Negative resistance converter.
a. Assemble the negative resistance converter shown in Figure 2 with $R_{1}=10 \mathrm{~K} \Omega, R_{2}=10 \mathrm{~K} \Omega$ and $R_{3}=4.7 \mathrm{~K} \Omega$.
b. Attach the adjustable +6 V DC source to the input of the converter. Using your DMM, measure the source current, $i_{s}$, at six equally spaced values of $v_{s}$ between 0 and +5 V . Plot these data as an I-V curve. Explain the result
c. Voltage-controlled current source
d. Modify the negative resistance converter of the previous part to form the voltage controlled current source shown in Figure 3. Leave $R_{1}=10 \mathrm{~K} \Omega$ and $R_{2}=10 \mathrm{~K} \Omega$. Select $R_{3}=1 \mathrm{~K} \Omega$. What is the value of $R_{S}$ that will make this a constant current source? Select that value.
e. Select $R_{L}=1 K \Omega$. Using your DMM, measure the load current, $i_{L}$, at six equally spaced values of $v_{S}$ between 0 and +5 V . Plot these data. Repeat with $R_{L}=470 \Omega$. What do you conclude about the capabilities of the constant current source?
5. Current-to-voltage converter.
a. Assemble the current-to-voltage converter circuit shown in Figure 7a. The collector of the phototransistor, which has the shorter of the two leads, connects to a +6 V DC source via a resistor of value $R_{p}=1 K \Omega$. The emitter of the transistor, which has the longer lead, connects to the inverting input of the op amp. The feedback resistor has value $R=10 \mathrm{~K} \Omega$.
b. The phototransistor responds to light in the visible range by generating a current, which the I-V converter converts to a voltage. In this experiment, the light will come from an LED which you will connect to the function generator via a resistor of value $R_{D}=1 K \Omega$. The packages of the LED and the phototransistor look alike, but you can easily tell them apart if you look at them from the top. As shown in Figure 7b, the phototransistor has a small black square in the middle, whereas the LED has a small dot whose color will depend on the color of the LED you have. Set the function generator to produce $v_{s}$, a 3 V p-p square-wave at 4 Hz. You should see the LED blink. To get the maximum response from the circuit, bend the leads of the phototransistor and the LED at $90^{\circ}$ and orient the devices so that the tops of the two devices are pointing to each other, like two gentlemen bowing deeply to each other, with only a millimeter of separation between them.
c. Apply power to the circuit. Observe $v_{s}$ on Ch1 of the oscilloscope and $v_{\text {out }}$ on Ch2. What do you see on Ch2? How fast does the phototransistor respond to a change in the light? Increase the amplitude of $v_{S}$ to $5 \mathrm{~V} p-\mathrm{p}$. What happens? Change the frequency of the square wave to 100 Hz . Can the phototransistor follow?
d. To test whether the phototransistor is responding to light or perhaps to an induced electrical signal from the LED circuit, place an opaque object like your plastic student ID card between the LED and the phototransistor. What happens?


Figure 7: Phototransistor and LED circuit

